Calculus III — MTH 253 November 24, 2005

Midterm 2

Name : ____________________________ **Score :** (/140) = ______%

Problem 1. Find the equation for a sphere that passes through the point $(4, 3, -1)$ and has its center at $(3, 8, 1)$.

(/10)

Problem 2. If $\mathbf{a} = 2\,\mathbf{i} - 3\,\mathbf{j}$, and $\mathbf{b} = \mathbf{i} + 5\,\mathbf{j}$,

a) Find $|\mathbf{a}|$

(/5)

b) Find $3\,\mathbf{a} + 4\,\mathbf{b}$

(/5)

Problem 3. For $\mathbf{a} = \mathbf{i} - 2\,\mathbf{j} + 3\,\mathbf{k}$, $\mathbf{b} = 5\,\mathbf{i} + 9\,\mathbf{k}$,

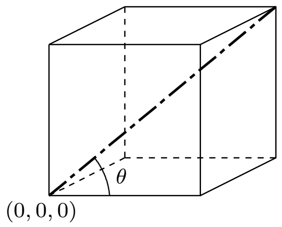
a) Find $\mathbf{a} \cdot \mathbf{b}$

(/5)

b) Find the scalar and vector projections of $\mathbf{b}$ onto $\mathbf{a}$.

(/5)

Problem 4. Find the angle between the main diagonal of a unit-cube and one of it's edges, to the nearest degree.



(/10)

Problem 5. Given the points $P(1,0,0)$, $Q(0,2,0)$, and $R(0,0,3)$:

a) Find a vector orthogonal to the plane containing P, Q, and R.

(/10)

b) What is the area of triangle $\triangle PQR$?

(/10)

Problem 6. Write a parametric and symmetric equations of a line passing through the points $(3,1,-1)$ and $(3,2,-6)$.

(/20)

Problem 7. Write the equation of the plane which contains the point $(6,3,2)$ and is perpendicular to the vector $\langle -2,1,5\rangle$.

(/10)

Problem 8. Find the point at which the line $x = 1 + 2\,t$, $y = -1$, $z = t$ intersects the plane $2\,x - y + z - 5 = 0$.

(/10)

Problem 9. State the *binomial theorem*, and then use it to expand $f(x) = \sqrt[3]{1 + x^2}$ as a power series, writing the individual terms up to degree 3.

(/10)

Problem 10. Find the fourth degree *Taylor approximation*, $T_4(x)$, for the function $f(x) = \cos x$ centered at the point $a = \frac{\pi}{3}$. What is the maximum error, $R_4(x)$, on the interval $\left[0, \frac{2\pi}{3}\right]$?

(/10)