Report. Solving linear viscoelastic model with GetFEM

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March 4, 2022

Abstract

This report describes how to solve linear viscoelastic equation with GetFEM

I. Linear elastic equations

$$
\begin{aligned}
div(\overline{\sigma}(\mathbf{u})) &= 0 \text{ in } \Omega, & (1)\\
\overline{\sigma}(\mathbf{u}) &= G\nabla\mathbf{u} + \frac{G}{1-2\nu}\nabla.\mathbf{u} & (2)
\end{aligned}
$$

II. Linear viscoelastic equations

Generalized maxwell model with one relaxation mode (Zener Model)

$$
\begin{aligned}
G(t) &= G_\infty + (G_0 - G_\infty)e^{-\beta t} & (3)\\
\overline{\sigma}(\mathbf{u(t)}) &= \int_0^t G(t-s)\frac{\partial}{\partial s}\big(\nabla\mathbf{u}(s) + \frac{1}{1-2\nu}\nabla.\mathbf{u}(s)\big)ds & (4)
\end{aligned}
$$

Integrating by part eq.(4) with deformation startng at $t = 0$ gives

$$
\begin{aligned}
\overline{\sigma}(t) &= G(0)\nabla\mathbf{u}(t) + \frac{G(0)}{1-2\nu}(\nabla.\mathbf{u}(t)) + \int_0^t G'(t-s)\big(\nabla\mathbf{u}(s) + \frac{1}{1-2\nu}\nabla.\mathbf{u}(s)\big)ds & (5)\\
G'(t) &= -\beta(G_0 - G_\infty)e^{-\beta t} & (6)
\end{aligned}
$$

Eq.(5) can be written as

$$\overline{\sigma}(t) = \overline{\sigma}^e(t) + \overline{\sigma}^{visc}(t) \quad (7)$$

with

$$
\begin{aligned}
\overline{\sigma}^e(t) &= G(0)H(t) & (8)\\
H(t) &= \nabla\mathbf{u}(t) + \frac{1}{1-2\nu}\nabla.\mathbf{u}(t) & (9)\\
\overline{\sigma}^{visc}(t) &= -\beta(G_0 - G_\infty)\int_0^t e^{\beta(s-t)}H(s)ds & (10)
\end{aligned}
$$

Define the time step $\Delta t = t_{n+1} - t_n$ from the previous time instant t_n to the current time t_{n+1}, eq (10) can be rewritten as

$$\overline{\sigma}^{visc}(t) = \overline{\sigma}^{visc}(t_n) - \beta(G_0 - G_\infty)\int_{t_n}^{t_{n+1}} e^{\beta(s-t^{n+1})}H(s)ds \quad (11)$$

$$\overline{\sigma}^{visc}(t) = \overline{\sigma}^{visc}(t_n) - \beta(G_0 - G_\infty)\frac{\Delta t}{2}(H(t_{n+1}) + e^{-\beta\Delta t}H(t_n)) \quad (12)$$

References

[Depascalis, 2014] De Pascalis, R., et all. On nonlinear viscoelastic deformations *Proceeding of the Royal Society*, 2014.