Detection in ISI Channels

Achilleas Anastasopoulos

September 28, 2006

Consider an M-ary modulation defined by the set of signals $\{p_i(t)\}_{i=1}^M$, so that the transmitted signal is

$$s(t) = \sum_k p_{a_k}(t - kT), \tag{1}$$

where T is the symbol duration, the sequence $\{a_k\}_k$ is either memoryless or generated by a trellis (as in the case of CPM) and $a_k \in \{1, 2, \ldots, M\}$.[1]

The signal is transmitted through a channel $h(t)$ and observed in AWGN so that

$$r(t) = s(t) \otimes h(t) + n(t) = \sum_k p_{a_k}(t - kT) \otimes h(t) + n(t) = \sum_k f_{a_k}(t - kT) + n(t), \tag{2}$$

where $f_i(t) = p_i(t) \otimes h(t)$. Thus the overall result of the ISI channel is that we now have to detect the modulation given by the signal set $\{f_i(t)\}_{i=1}^M$.

The optimal receiver (one possible realization of it) should consist of a bank of symbol-spaced sampled matched filters matched to each of $f_i(t)$, that provide sufficient statistics for detection in the form of the M-dimensional vectors $\mathbf{r}_k = [r_{k,1}, \ldots, r_{k,M}]^T$, where

$$r_{k,i} = \int r(t) f_i(t - kT) dt = \sum_m \int f_{a_m}(t - mT) f_i(t - kT) dt + n_{k,i} \tag{3}$$

$$= \sum_m \int f_{a_m}(t + (k - m)T) f_i(t) dt + n_{k,i} = \sum_l \int f_{a_{k-l}}(t + lT) f_i(t) dt + n_{k,i} \tag{4}$$

$$= \sum_l R_{a_{k-l},i}(l) + n_{k,i}, \tag{5}$$

where we have used the notation $R_{i,j}(l) = \int f_i(t + lT) f_j(t)$, and $n_{k,i}$ is AGN with correlation given by

$$E\{n_{k,i} n_{m,j}\} = \frac{N_0}{2} \int f_i(t - kT) f_j(t - mT) dt = \frac{N_0}{2} R_{i,j}(m - k), \tag{6}$$

so that $E\{\mathbf{n}_k \mathbf{n}_m^T\} = \frac{N_0}{2} \mathbf{R}(m - k)$, with $\mathbf{R}(l) = [R_{i,j}(l)]$.

[1] A special case of the above model is linear modulations where $p_{a_k}(t) = b_k p(t)$, but in the following we will assume the more general model.

In vector form the discrete-time model after matched filtering becomes

$$\mathbf{r}_k = \sum_l \mathbf{R}_{a_{k-l}}(l) + \mathbf{n}_k, \tag{7}$$

where $\mathbf{R}_j(l) = [R_{j,1}(l), \ldots, R_{j,M}(l)]^T$.

We can now perform noise whitening resulting in the N-dimensional discrete-time model

$$\mathbf{z}_k = \sum_l \mathbf{F}_{a_{k-l}}(l) + \mathbf{w}_k, \tag{8}$$

where $\mathbf{w}_k$ is AWGN with correlation matrix $E\{\mathbf{w}_k \mathbf{w}_m^T\} = \frac{N_0}{2}\mathbf{I}_N \delta_{k-m}$.

Clearly the model in (8) can be thought of as a non-linear vector ISI model and can be represented by an augmented trellis modeling the ISI and the inherent memory in the sequence $\{a_k\}_k$ (in the case of modulations with memory, such as CPM).

In the special case of linear modulations (see previous footnote) we end up with a linear vector ISI model of the form

$$\mathbf{z}_k = \sum_l b_l \mathbf{F}(l) + \mathbf{w}_k. \tag{9}$$

In order to implement in GnuRadio ISI equalization for a given modulation and channel, the equivalent discrete-time vector model has to be identified. Once this is done, the required lookup table should be constructed according to (8). For instance if $F_i(l) \neq 0$ for all $l \in \{0, 1, \ldots, L-1\}$, then we end up with a lookup table with M^L entries (representing all possible combinations of symbols $[a_k, \ldots, a_{k-L+1}]$), each of which is the N-dimensional vector $\sum_{l=0}^{L-1} \mathbf{F}_{a_{k-l}}(l)$.